

TRIGONOMETRIE

Définitions

A tout réel θ on associe l'unique point M du cercle trigonométrique tel que θ soit une mesure de l'angle $(\vec{u}, \overrightarrow{OM})$. Alors :

- $\cos \theta$ est l'abscisse du point M .
- $\sin \theta$ est l'ordonnée du point M .
- $\tan \theta = \frac{\sin \theta}{\cos \theta}$ et $\cotan \theta = \frac{\cos \theta}{\sin \theta}$.

Formules de base

$$\cos^2 \theta + \sin^2 \theta = 1 \quad \frac{1}{\cos^2 \theta} = 1 + \tan^2 \theta \quad \frac{1}{\sin^2 \theta} = 1 + \cotan^2 \theta$$

Lignes trigonométriques usuelles

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	
$\cotan \theta$		$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Symétries

$$\begin{array}{lll} \cos(-\theta) = \cos \theta & \sin(-\theta) = -\sin \theta & \tan(-\theta) = -\tan \theta \\ \cos(\pi - \theta) = -\cos \theta & \sin(\pi - \theta) = \sin \theta & \tan(\pi - \theta) = -\tan \theta \\ \cos(\pi + \theta) = -\cos \theta & \sin(\pi + \theta) = -\sin \theta & \tan(\pi + \theta) = \tan \theta \\ \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta & \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta & \tan\left(\frac{\pi}{2} - \theta\right) = \cotan \theta \\ \cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta & \sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta & \tan\left(\frac{\pi}{2} + \theta\right) = -\cotan \theta \end{array}$$

Formules d'addition

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Formules de duplication

$$\cos 2a = \cos^2 a - \sin^2 a = 2 \cos^2 a - 1 = 1 - 2 \sin^2 a$$

$$\sin 2a = 2 \sin a \cos a$$

$$\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}$$

Transformation de produits en sommes (linéarisation)

$$\cos a \cos b = \frac{1}{2} [\cos(a + b) + \cos(a - b)] \qquad \cos^2 a = \frac{1}{2} (1 + \cos 2a)$$

$$\sin a \sin b = -\frac{1}{2} [\cos(a + b) - \cos(a - b)] \qquad \sin^2 a = \frac{1}{2} (1 - \cos 2a)$$

$$\sin a \cos b = \frac{1}{2} [\sin(a + b) + \sin(a - b)] \qquad \sin a \cos a = \frac{1}{2} \sin 2a$$